

ST NICHOLAS OF TOLENTINE CALCULATION POLICY

~choosing the best representation for the maths~

NB: Pictorial can be a representation of the concrete where possible

Numberlines are invaluable as they help to secure their understanding of numberlines ready for use across the four operations.

Each time you include a new PV column, model which equipment represents this and how to draw it (eg Hundreds in y3, Thousands in y4 etc)

EYFS - Much of the learning will be through manipulatives and concrete objects, with some children moving onto drawing representations. However, the language is crucial at this stage to set a good foundation for KS1. Introducing chn to the more 'standard' concrete resources such as rekenrek, ten frames or Hungarian frames and dienes will be very beneficial!	
MATCH SORT COMPARE Same different sort set more fewer Bigger smaller larger taller shorter heavier lighter Holds more/ holds less Capacity pattern	COUNTING Count altogether subitise times Made up of part whole
COMPARING The ____ are the same The ____ are different The ____ and the ____ match The ____ has the same capacity as ____	I counted ____ There is 1 . ____ There are 2/3 . ____ There are ____ altogether. I can see.. I can see ____ without counting. I can subitise ____ We clapped ____ times
SORTING The ____ and the ____ are a set/not a set because ____ I have sorted the objects by ____ All my objects are ____ Another way to sort the objects are ____ This belongs/doesn't belong in the set because ____	There are ____ There are ____ altogether. ____ is 1 more/less than ____

The rule is _____

This set has more/fewer than _____

PATTERNS

My pattern is _____, _____, _____

The _____ comes next in my pattern

I have used _____ to make my pattern.

Next, I need _____ to continue my pattern.

I need _____ to finish my pattern.

The _____ is in the wrong place

I can see _____ is made up of _____ and _____

I can see _____ is made up of _____, _____ and _____

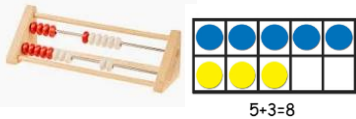

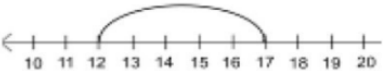
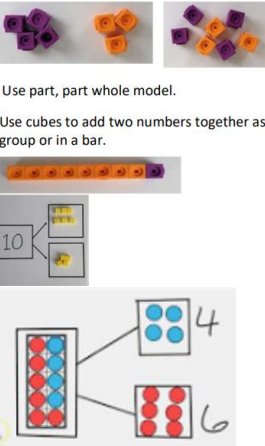
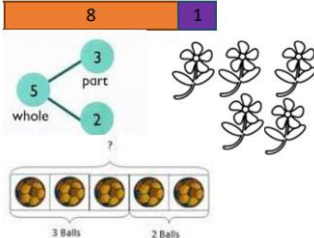

There is _____ here and _____ there so there must be _____ altogether.

_____ is a part and _____ is a part.

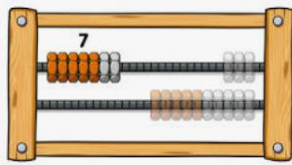
If _____ is a part, then the other part must be _____

_____ is a part of _____

The whole is _____

YEAR 1 ADDITION VOCABULARY: part whole add sum equals count on tens ones plus add more than				
Objective	Concrete (representation) Multilink, counters, bead strings, tenframes rekenrek	Pictorial (process) Part whole model Bar model numberline	Abstract Number sentence	Sentence stems
Counting in ones	 <p>5+3=8</p> <p>Start with the larger number on the frame and then count on to the smaller number one by one.</p>	<p>Use a number line to count on in ones.</p> 	$5 + 3 = 8$ $8 = 5 + 3$	<p>The last number I said was _____, so there are _____ objects in total.</p> <p>I need to start counting from _____</p> <p>The number that comes after _____ is _____</p> <p>I will say the number _____ because ...</p> <p>I will not say the number _____ because ...</p>
Starting at the bigger number and counting on	<p>As above</p> <p>Encourage children to recognise 5 on the rekenrek or ten frame and the fact they know it so they don't need to count from 1</p>	 <p>12 + 5 = 17</p> <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	$5 + 12 = 17$ Place the larger number in your count on the smaller number to answer.	<p>Addend plus addend equals sum. Sum equals addend plus addend.</p> <p>I start at _____ and then count on _____</p>
PLACE VALUE: Combining 2 parts to make a part whole model	 <p>Use part, part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>	<p>Use pictures to add two numbers together as a group or in a bar.</p> 	$8 = 5 + 3$ $5 + 3 = 8$  <p>Use the part part whole diagram as shown above to move into the abstract.</p> <p>Include missing number questions to support varied fluency:</p> $8 = ? + 3$ $5 + ? = 8$	<p>There are _____ in the whole group. There are _____ in this part of the group.</p> <p>_____ is the whole; _____ is a part and _____ is a part.</p> <p>The whole is _____ and one part is _____ so the other part must be _____.</p>

Regrouping to make 10. This is an essential skill for column addition later.

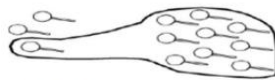


$$7 + 3$$

Start with the bigger number and use the smaller number to make ten first

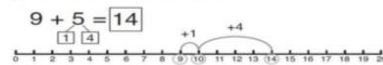


$6 + 4 = 10$ then $+ 1$ to make 11



$$3 + 9 =$$

Use pictures or a number line. Regroup or partition the smaller number using the part, part whole model to make 10.



$$7 + 4 = 11$$

If I am at seven, how many more do I need to make 10? How many more do I add on now?

This is ten ones. It is also one ten

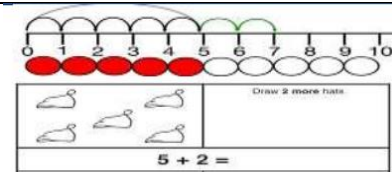
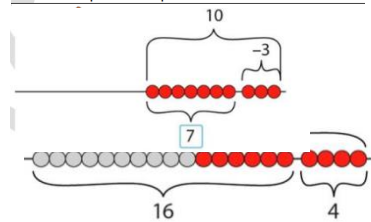
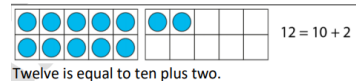
___ is five and ___ more.
___ is equal to ten plus ___.

I know that ___ plus ___ is equal to ten so ___ plus ___ is equal to ___.

I know that $6 + 4 = 10$ so I know that $16 + 4 = 20$
(eg $6 + 4 = 10$ so $16 + 4 = 20$)

Represent & use number bonds and related subtraction facts within 20

3 more than 7



$$7 - 3 = 4$$

Missing number questions:

$$10 = ? + 3$$

$$7 + ? = 10$$

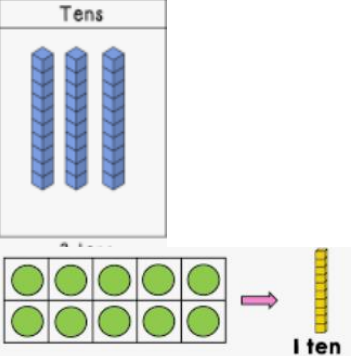
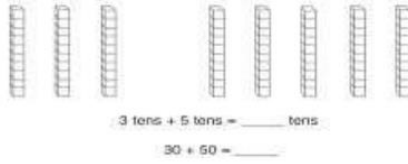
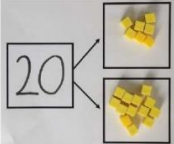
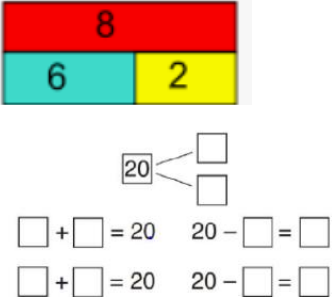
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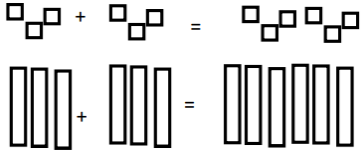
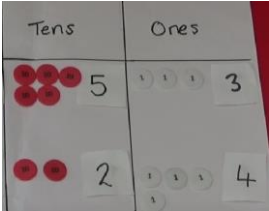
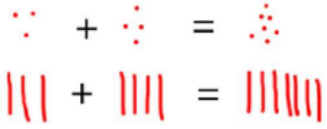
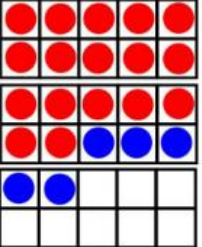
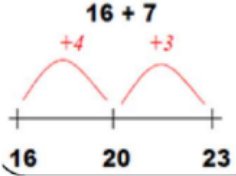

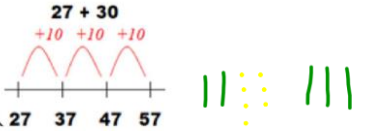
___ more than ___ is equal to ___.

___ more than ___ is ___.

___ is ___ more than ___.

I know that ___ minus ___ is equal to ten so ___ minus ___ is equal to ___.
(eg $10 - 4 = 6$ so $20 - 4 = 16$)

YEAR 2 ADDITION	NEW VOCABULARY: partition bar model multiple of ten Fact family total inverse commutative count in steps count in multiples estimate compare sum digit addend subtrahend minuend			
Objective	Concrete , tenframes dienes/base ten place value counters Place value chart	Pictorial Drawn representations for base ten Bar models Part whole models numberlines	Abstract Number sentence Missing number problems Column method partition	Sentence stems
Adding multiples of ten		 <p>Use representations for base ten.</p>	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$	<p>I know ___ + ___ is ___ so I know ___ tens + ___ tens is ___ tens (eg $3+2 = 5$ so $30+50= 80$)</p>
Use known number facts			<p>Explore commutativity of addition by swapping the addends to build a fact family. Explore the concept of the inverse relationship of addition and subtractions and use this to check calculations.</p> $\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$	<p>I know that ___ plus ___ is equal to ten so ___ plus ___ is equal to ___. I know that $6+4= 10$ so I know that $16+4 = 20$ (eg $6+4 = 10$ so $16+4=20$)</p> <p>I know that ___ minus ___ is equal to ten so ___ minus ___ is equal to ___. (eg $10 - 4 = 6$ so $20-4=16$)</p>

	  $53+24$	 $3 + 4 = 7$ <i>leads to</i> $30 + 40 = 70$ <i>leads to</i> $300 + 400 = 700$	<p>I know that ___ plus ___ is equal to ten so ___ plus ___ is equal to ___.</p> <p>I know that $6+4= 10$ so I know that $16+4 = 20$ (eg $6+4 = 10$ so $16+4=20$)</p> <p>I know that ___ minus ___ is equal to ten so ___ minus ___ is equal to ___.</p> <p>(eg $10 - 4 = 6$ so $20-4=16$)</p>
<p>Add a two digit number and ones</p>	 <p>eg: $17+5$ Use the ten frame to make ten</p> <p>Explore the pattern: $17+5$ $27+ 5$ $37+5$ etc</p>	 <p>$16 + 7$</p> <p>Use a numberline to model making the next ten and then adding the remainder of the number.</p> <p>Continue to explore the pattern: $16+ 7$ $26+ 7$ $36+ 7$ etc</p>	<p>$16+ 7$</p> <p>$16 + 4 = 20$ $20+ 3 = 23$</p> <p>___ can be partitioned into ___ and ___</p> <p>The multiple of 10 after _____ is _____</p> <p>I need to add _____ to get to the next 10</p> <p>_____ + _____ = _____</p> <p>I need to add _____ more.</p> <p>So _____ + _____ = _____</p>
<p>Add a 2 digit number and tens</p>	 <p>$25 + 10 = 35$</p> <p>Explore that the ones digit does not change</p>	 <p>$27 + 30$</p> <p>Use a numberline to model adding ten (to add multiples of ten)</p> <p>Draw the dienes and count the tens aloud as they are added.</p>	<p>$27 + 10 = 37$</p> <p>$27 + 20 = 47$</p> <p>$27 + \square = 57$</p> <p>How many tens do I need to add to 34 to reach 74?</p>

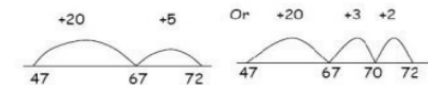
Add 2 two-digit numbers

Partition the number into tens and ones using dienes (or PV counters when chn are secure with dienes)



T	O

NB: dienes are easier to draw in the pictorial stage than PV counters



Use number line and bridge ten using part whole if necessary.

$$\begin{array}{r} 25 + 47 \\ \swarrow \quad \searrow \\ 20 + 5 \quad 40 + 7 \end{array}$$

$$20 + 40 = 60$$

$$5 + 7 = 12$$

$$60 + 12 = 72$$

Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values.

2	5			
4	7	+		
12		(5 + 7)		
60		(20 + 40)		
72		(60 + 12)		

I need to add ___ tens and ___ ones.

I start at ___.

First I add ___ tens. Then I add ___ ones.

Add three 1-digit numbers

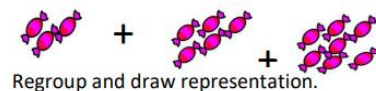


Combine to make 10 first if possible, or bridge 10 then add third digit



$$4 + 7 + 6 = 17$$

Put 4 and 6 together to make 10. Add on 7.



Regroup and draw representation.

$$4 + 7 + 6 = 17$$

$$\begin{array}{l} (4 + 7 + 6) = 10 + 7 \\ = 17 \end{array}$$

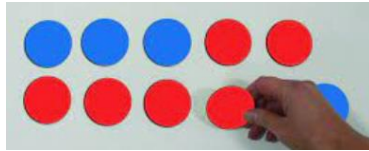
Combine the two numbers that make/bridge ten then add on the third.

___ and ___ make 10. If I add ___ more, the total is ___

10 + ___ = ___ So ___ + ___ + ___ = ___

Estimate the answers to questions and use inverse operations to check answers

Use double sided counters to show the inverse of addition and subtraction



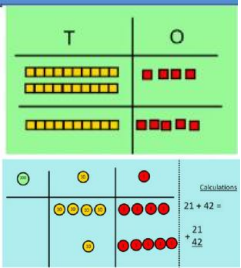
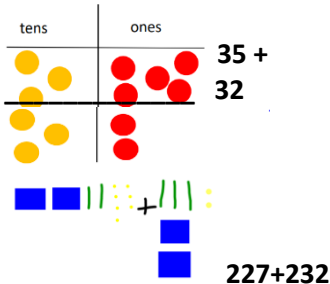
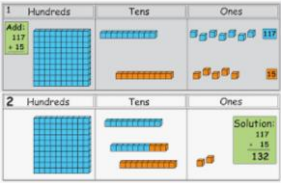
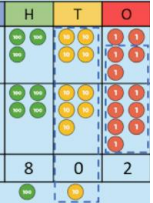
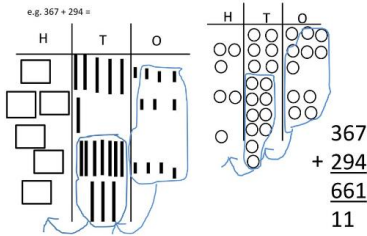
Use numberlines to illustrate estimation and also to model the relationship between addition and subtraction



If _____ is a part and _____ is a part, then _____ is the whole.

If _____ is the whole and _____ is a part, then _____ is the other part.

The inverse of _____ is _____

YEAR 3 ADDITION	NEW VOCABULARY: column addition formal written method regroup exchange Hundreds estimate expression equation			
Objective	Concrete tenframes rekenrek dienes/base ten place value counters place value grid	Pictorial Drawn representations for base ten Bar models Part whole models	Abstract Number sentence Missing number problems Column method	Sentence stems
Column Addition—no regrouping Add two or three 2 or 3digit numbers.	Use Dienes or PV counters to add (up to Hundreds) 	Children to draw dienes or PV counters 	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ Add the ones first, then the tens, then the hundreds.	____ hundreds added to ____ hundreds is equal to ____ hundreds. I put ____ in the ____ column because ... ____ ones plus ____ ones is equal to ____ ones. ____ tens plus ____ tens is equal to ____ tens. ____ hundreds plus ____ hundreds is equal to ____ hundreds. ____ hundreds, ____ tens and ____ ones is equal to ____
Column Addition with regrouping.	Use dienes or PV counters  	3 digit addition regrouping in tens and ones drawing Dienes/place value counters e.g. $367 + 294 =$ 	$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ Start by partitioning the numbers before formal column to show the exchange. $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$	____ tens + ____ tens = ____ tens If I have ____ tens, I can exchange them for ____ hundred and ____ tens. I have ____ hundreds, ____ tens and ____ ones, so altogether I have ____ In column addition, we start from the place value column that has the ____ value. The ____ is in the ____ column. It represents ____

**YEAR 4 -6
ADDITION**

NEW VOCABULARY:
 y4 thousands 4 digit number operation
 Y5 ten thousands hundred thousands million
 tenths hundredths powers of ten place holder
 y6 ten million thousandths

Objective

Concrete
 dienes/base ten place value
 counters place value grid

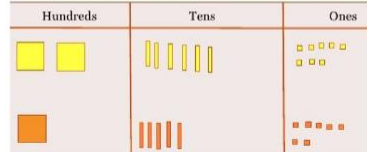
Pictorial
 Drawn representations for base ten
 Bar models
 Part whole models
 Drawn PV frames and counters

Abstract
 Number sentence
 (calculation)
 Missing number problems
 Column method

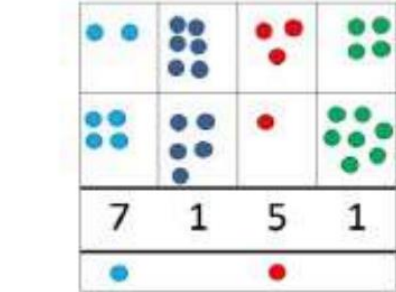
Sentence stems

**Y4—add
 numbers with
 up to 4 digits**

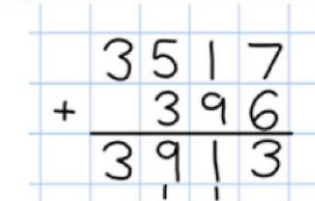
Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.



Use money to represent



Draw representations using place value grid.

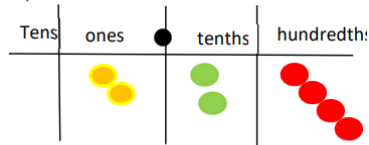


Continue from previous work to carry hundreds as well as tens.
 Relate to money and measures.

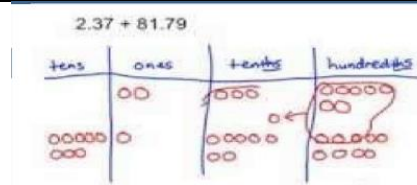
____ ones added to ____ ones is equal to ____ ones.
 ____ plus ____ plus the 1 that I exchanged from the last column is equal to ____
 I have ____ hundreds/tens/ones, so I do/do not need to make an exchange.
 In column addition, we start from the place value column that has the ____ value.
 The ____ is in the ____ column. It represents ____

Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money

As year 4



Introduce decimal place value counters and model exchange for addition.



$$\begin{array}{r}
 72.8 \\
 + 54.6 \\
 \hline
 127.4 \\
 11
 \end{array}$$

$$\begin{array}{r}
 \pounds 23.59 \\
 + \pounds 7.55 \\
 \hline
 \pounds 31.14
 \end{array}$$

Insert zeros for place holders.

$$\begin{array}{r}
 81,059 \\
 3,668 \\
 15,301 \\
 + 20,551 \\
 \hline
 120,579 \\
 \\

 \end{array}$$

$$\begin{array}{r}
 23.361 \\
 9.080 \\
 59.770 \\
 + 1.300 \\
 \hline
 93.511 \\
 21.211
 \end{array}$$

In column addition, we start from the place value column that has the ____ value.

The ____ is in the ____ column. It represents ____

I say ____ point ____ but I think ____ and ____ tenth(s).

I say ____ point ____ but I think ____ and ____ hundredths.

Add decimal numbers up to 3 decimal places



Insert zeros for place holders.

$$\begin{array}{r}
 81,059 \\
 3,668 \\
 15,301 \\
 + 20,551 \\
 \hline
 120,579 \\
 \\

 \end{array}$$


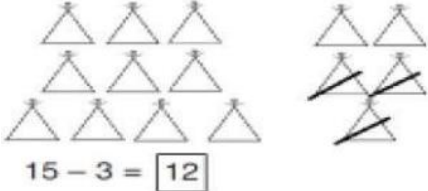


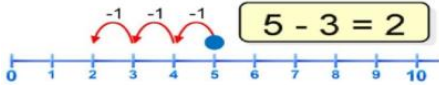
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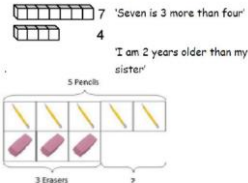
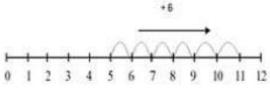
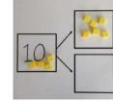
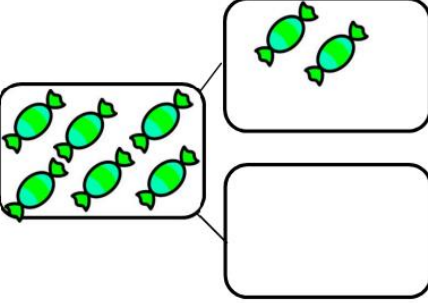
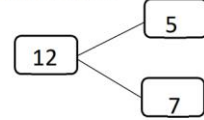

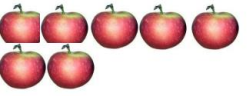
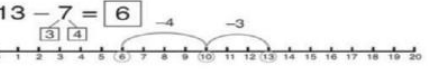

____ added to ____ is equal to ____

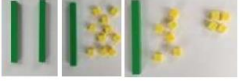


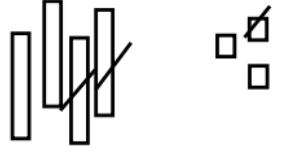
____ subtract ____ is equal to ____

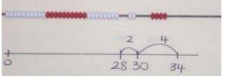
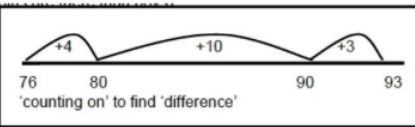
____ tenths added to ____ tenths is equal to ____ tenths.



I do/do not need to make an exchange because ...


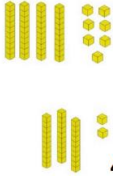
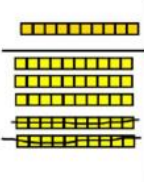
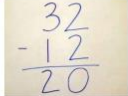
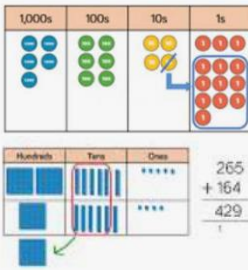
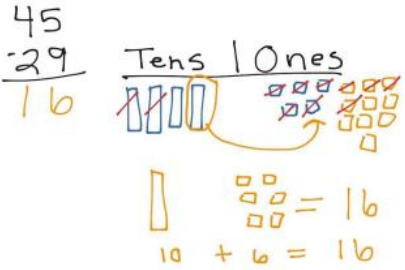
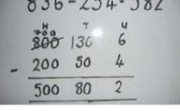
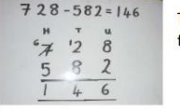
YEAR 1 SUBTRACTION		VOCABULARY: part whole taking away counting back find the difference bond to ten		
Objective	Concrete (representation) Multilink, counters, bead strings, tenframes rekenrek	Pictorial (process) Part whole model Bar model numberline	Abstract Number sentence	Sentence stems
Taking away ones	Use physical objects to show how objects can be taken away 	Draw objects and cross out objects to show which have been taken away 	$7 - 4 = 3$ $16 - 9 = 7$ One less than....	AUGMENTATION: First there were _____ Then _____ were taken away. Now there are _____
Counting back	 Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?	I need to start from _____ I need to make _____ jumps backwards. I land on _____ This means that _____ - _____ = _____

<p>Find the difference</p>	<p>Compare objects and amounts</p>  <p>7 'Seven is 3 more than four' 4 'I am 2 years older than my sister' 5 Pencils 3 Erasers 7 Lay objects to represent bar model.</p>	<p>Count on using a number line to find the difference.</p> 	<p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?</p>			
<p>Represent and use number bonds and related subtraction facts within 20</p> <p>Include subtracting zero</p> <p>Part Part Whole model</p>	<p>Link to addition. Use PPW model to model the inverse.</p>  <p>If 10 is the whole and 6 is one of the parts, what is the other part?</p> $10 - 6 = 4$	 <p>Use pictorial representations to show the part.</p>	<p>Move to using numbers within the part whole model.</p>  <p>Include missing number problems: $12 - ? = 5$ $7 = 12 - ?$</p>	<p>If the whole is _____ and _____ is a part, then the other part is _____</p> <p>_____ minus _____ is _____</p> <p>_____ - _____ = _____</p>		
<p>Make 10</p>	<p>14 - 9</p>  <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>  $5 - 2 = 3$	<p>13 - 7 = 6</p>  <p>13 - 7</p> <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p>  <p>Draw pictures to represent</p>	<p>16 - 8</p> <p>How many do we take off first to get to 10? How many left to take off?</p> <table border="1" data-bbox="1288 933 1601 997"> <tr> <td>8</td> <td>2</td> </tr> </table> <p>10 = 8 + 2 10 = 2 + 8 10 - 2 = 8 10 - 8 = 2</p>	8	2	<p>First I need _____ to reach 10 Then I need _____ to reach _____</p> <p>_____ subtract _____ = _____</p> <p>_____ and _____ is a bond to 10 10 is made up of _____ and _____</p>
8	2					

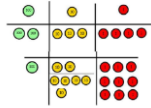
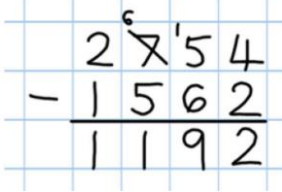
YEAR 2 SUBTRACTION		VOCABULARY: part whole taking away counting back find the difference Partitioning addend subtrahend minuend		
Objective	Concrete (representation) Multilink, counters, bead strings, tenframes rekenrek	Pictorial (process) Part whole model Bar model numberline	Abstract Number sentence	Sentence stems
Regroup a ten into ten ones	 <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>	 $20 - 4 =$	$20 - 4 = 16$	To subtract ____ ones, I need to subtract 1 ____ times.
Partitioning to subtract without regrouping. 'Friendly numbers'	$34 - 13 = 21$  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	<p>Children draw representations of Dienes and cross off.</p>  $43 - 21 = 22$	$43 - 21 = 22$	First I take ____ tens and then I take ____ ones. I have ____ tens and ____ ones left. I have ____ left.

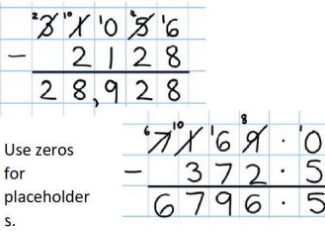
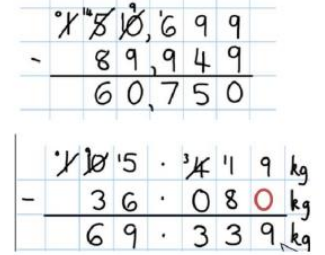
<p>Make ten strategy Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</p>	 <p>34-28</p> <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>	 <p>Use a number line to count on to next ten and then the rest.</p>	<p>93-76=17</p>	<p>I need to subtract _____ to get to 10 I can partition _____ into _____ and _____ I need to subtract _____ more. _____ less than _____ is _____</p>
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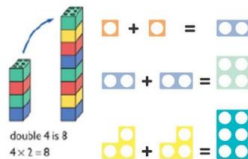


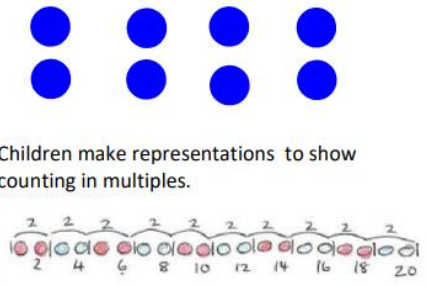
<p>YEAR 3 SUBTRACTION VOCABULARY: part whole taking away counting back find the difference Partitioning exchange</p>				
Objective	Concrete (representation) Multilink, counters, bead strings, tenframes rekenrek	Pictorial (process) Part whole model Bar model numberline	Abstract Number sentence	Sentence stems
<p>Subtract numbers mentally, including: three digit number - ones three digit number - tens three digit number</p>	 <p>Use PV counters or dienes on a PV grid to make the number and remove ones or tens</p>	 <p>Draw the counters on a grid</p>	<p>Vary the position of the answer and question. Expose children to missing number questions and vary the missing part of the calculation. 678 = ? - 1 688 - 10 = ? 678 = ? - 100</p>	<ul style="list-style-type: none"> When adding or subtracting 1s to or from a number, the digit in the _____ column always changes. If I know 3 + 6 = 9, then I know that 123 + 6 = _____

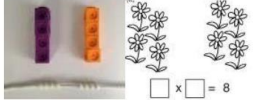

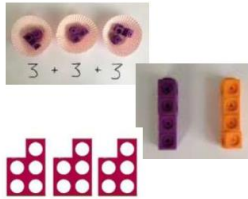
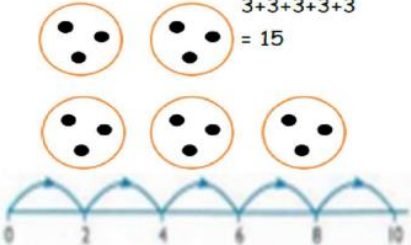

	 <p>Move to representing counters with digits</p>			
<p>Column subtraction without regrouping (friendly numbers)</p>	 <p>47-32</p>	 <p>Draw representations to support understanding</p>	<p>Calculations</p> $47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ <p>Intermediate step may be needed to lead to clear subtraction understanding.</p> 	<ul style="list-style-type: none"> _____ ones/tens/hundreds minus _____ ones/tens/hundreds is equal to _____ ones/tens/hundreds. Now there are _____ hundreds, _____ tens and _____ ones. The answer is _____ <p>I put the minuend on top I start from the column with the smallest value. I subtract FROM the minuend</p>
<p>Column subtraction with regrouping</p>	 <p>Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten</p>	 <p>Children may draw base ten or PV counters and cross off.</p>	<p>836-254=582</p>  <p>Begin by partitioning into pv columns</p> <p>728-582=146</p>  <p>Then move to formal method.</p>	<ul style="list-style-type: none"> _____ ones subtract _____ ones is equal to _____ ones. I will exchange 1 ten for _____ ones. Now I have _____ hundreds, _____ tens and _____ ones. The answer is _____


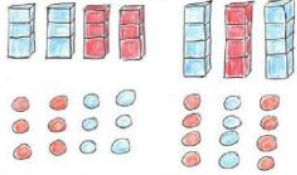
	into ten ones. Use the word exchange			
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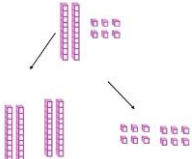
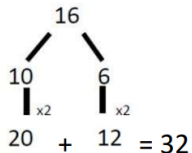
YEAR 4-6 SUBTRACTION		VOCABULARY: part whole taking away counting back find the difference Partitioning		
Objective	Concrete (representation) Multilink, counters, bead strings, tenframes rekenrek	Pictorial (process) Part whole model Bar model numberline	Abstract Number sentence	Sentence stems
Subtracting tens and ones Year 4 subtract with up to 4 digits. Introduce decimal subtraction through context of money	Start representing the exchange using dienes first then move on to PV counters 	Children to draw pv counters and show their exchange—see Y3	 <p>Use the phrase 'take and make' for exchange</p>	<ul style="list-style-type: none"> • ____ tens subtract ____ tens is equal to ____ • I will exchange 1 hundred to make ____ tens. • Now there are ____ hundreds, ____ tens and ____ ones. The answer is ____ • ____ hundreds subtract ____ hundreds is equal to ____ • I will exchange 1 hundred for ____ tens, then 1 ten for ____ ones.

<p>Year 5- Subtract with at least 4 digits, including money and measures. Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal Up to 3 decimal places</p>	<p>As Year 4</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 <p>Use zeros for placeholder s.</p>	
<p>Year 6—Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).</p>	<p>As Year 4</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>		

YEAR 1 MULTIPLICATION		Programme of Study specifies the following objectives, however it does not require the explicit teaching of the mathematical symbol of multiplication multiplication division arrays multiply divide		
Objective	Concrete (representation) Multilink, counters, bead strings, tenframes rekenrek	Pictorial (process) Part whole model Bar model numberline	Abstract Number sentence	Sentence stems
Doubling	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double numbers</p> <p>Double 4 is 8</p> 	'double 4 is 8'	<ul style="list-style-type: none"> Double ____ is ____ ____ + ____ = ____ This is double ____ ____ is/is not a double. I know this because ...
Counting in multiples (2s, 5s, 10s)	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p> 	<p>Children make representations to show counting in multiples.</p> 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30	<ul style="list-style-type: none"> There are ____ in each pair. There are ____ pairs. There are ____ in total. There are ____ equal groups of 2 There are ____ altogether.

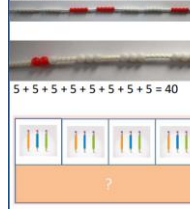
				<ul style="list-style-type: none"> • There are ____ groups of ten. There are ____ altogether. • There are ____ full ten frames. There are ____ in total. • There are ____ groups of 5 There are ____ altogether. • There are ____ 5s. There are ____ in total. • There are ____ 5s in 10
<p>Making equal groups and counting the total</p>	 <p>$\square \times \square = 8$</p> <p>Use manipulatives to create equal groups.</p>	<p>Draw  to show $2 \times 3 = 6$</p> <p>Draw and make representations</p>	<p>$2 \times 4 = 8$</p>	<ul style="list-style-type: none"> • The groups are equal/not equal because ... • There are ____ altogether. They can be put into equal groups of ____ There are ____ groups. • The ____ have/have not been shared equally. I know this because ... • There are ____ altogether. They are shared equally between ____ groups. There are ____ in each group. • There are ____ equal groups. There are ____ in each group. There are ____ altogether. • There are ____ groups of ____ ____ + ____ + ____ + ____ + ____ = ____
<p>Repeated addition</p>	 <p>$3 + 3 + 3$</p> <p>Use different objects to add equal groups</p>	<p>Use pictorial including number lines to solve prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p> <p>$3+3+3+3+3 = 15$</p> 	<p>Write addition sentences to describe objects and pictures.</p>  <p>$2+2+2+2+2 = 10$</p>	<ul style="list-style-type: none"> • There are ____ equal groups. There are ____ in each group. There are ____ altogether. • There are ____ groups of ____ ____ + ____ + ____ + ____ + ____ = ____

<p>Understanding arrays</p>	<p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p> 	<p>Draw representations of arrays to show</p>  <p>understanding</p>	<p>$2 \times 5 = 10$ $2 \times 3 = 6$</p>	<ul style="list-style-type: none"> • There are ____ rows. There are ____ in a row. There are ____ in total. • There are ____ columns. There are ____ in a column. There are ____ altogether.
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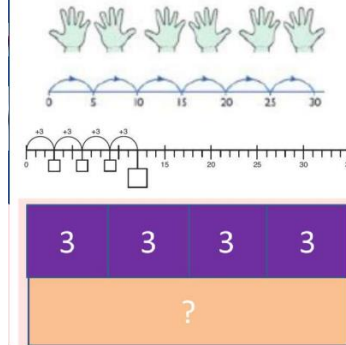
<p>YEAR 2 MULTIPLICATION</p> <p>Children should be able to recall and use multiplication and division facts for the 2, 5 and 10 times tables Children are introduced to the X symbol for multiplication Repeated addition times tables multiplication tables commutative</p> <ul style="list-style-type: none"> • There are 3 equal groups with ____ in each group. ____ + ____ + ____ = ____ ____ x ____ = ____ • ____ x ____ = ____ ____ multiplied by ____ is equal to ____ • ____ of ____ = ____ 				
<p>Objective</p>	<p>Concrete (representation) Multilink, counters, bead strings, tenframes rekenrek</p>	<p>Pictorial (process) Part whole model Bar model numberline</p>	<p>Abstract Number sentence</p>	<p>Sentence stems</p>
<p>Doubling</p>	<p>Model doubling using dienes and PV counters.</p>  <p>$40 + 12 = 52$</p>	<p>Draw pictures and representations to show how to double numbers</p>	<p>Partition a number and then double each part before recombining it back together.</p>  <p>$20 + 12 = 32$</p>	<ul style="list-style-type: none"> • Double ____ is ____ • Half of ____ is ____ • Double ____ is ____, so double ____ is ____ • Half of ____ is ____, so half of ____ is ____

Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)

Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.



Number lines, counting sticks and bar models should be used to show representation of counting in multiples.



Count in multiples of a number aloud.

Write sequences with multiples of numbers.

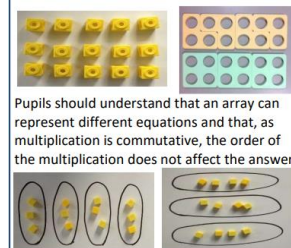
- 0, 2, 4, 6, 8, 10
- 0, 3, 6, 9, 12, 15
- 0, 5, 10, 15, 20, 25, 30

$$4 \times 3 = \square$$

- $\square \times 2$ is the same as \square lots of 2
- \square multiplied by 2 is equal to \square
- I know that $\square \times 2 = \square$, so I can add/subtract 2 to work out $\square \times 2$
- There are 3 equal groups with \square in each group. There are 3 groups of \square . $\square + \square + \square = \square$
- There are \square equal groups with \square in each group. There are \square groups of \square . There are \square altogether.

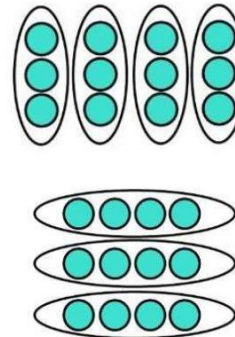
Multiplication is commutative

Create arrays using counters and cubes and Numicon.



Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.

Use representations of arrays to show different calculations and explore commutativity.



$$12 = 3 \times 4 \quad 12 = 4 \times 3$$

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

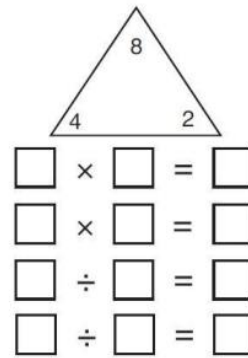
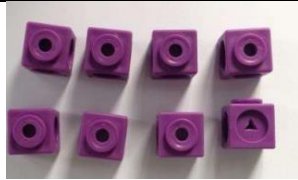
$$\square \times \square = \square$$

So $\square \times \square$ is \square

$$4 + 4 + 4 = 12$$

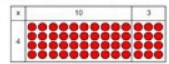
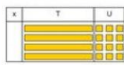

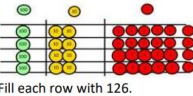
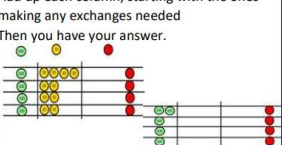
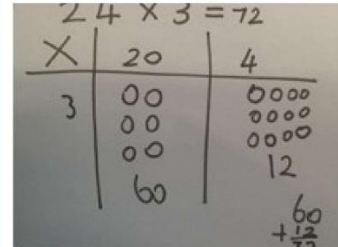
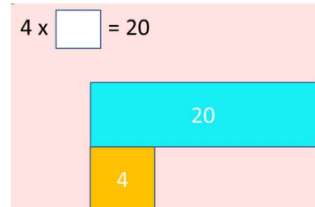
$$\text{So } 3 + 3 + 3 + 3 = 12$$

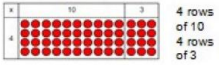
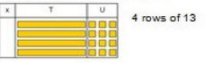
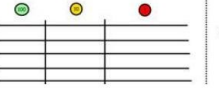
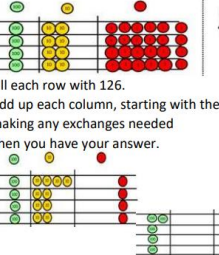
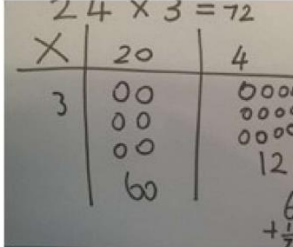
Using the Inverse This should be taught alongside division, so pupils learn how they work alongside each other.



$2 \times 4 = 8$ $4 \times 2 = 8$
 $8 \div 2 = 4$ $8 \div 4 = 2$
 $8 = 2 \times 4$ $8 = 4 \times 2$
 $2 = 8 \div 4$ $4 = 8 \div 2$
Show all 8 related fact family sentences

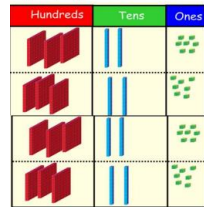
- _____ \times _____ ones is equal to _____ ones,
so _____ \times _____ tens is equal to _____ tens.
- _____ \div _____ is equal to _____,
so _____ tens \div _____ is equal to _____ tens.

YEAR 3 MULTIPLICATION		Children should be able to recall and use multiplication facts for the 3,4, and 8 times tables Derived facts missing number problems product								
Objective	Concrete (representation) Multilink, counters, bead strings, tenframes rekenrek	Pictorial (process) Part whole model Bar model numberline	Abstract Number sentence	Sentence stems						
Grid method, progressing to the formal method Multiply 2 digit numbers by 1 digit numbers	<p>Show the links with arrays to first introduce the grid method.</p>  <p>4 rows of 10 4 rows of 3</p> <p>Move onto base ten to move towards a more compact method.</p>  <p>4 rows of 13</p> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculations 4 x 126</p>  <p>Calculations 4 x 126</p> <p>Fill each row with 126. Add up each column, starting with the ones making any exchanges needed Then you have your answer.</p> 	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p>  <p>Bar model are used to explore missing numbers</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1344 598 1568 662"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p>$210 + 35 = 245$</p> <p>Move forward to the formal written method:</p> $\begin{array}{r} 35 \\ \times 7 \\ \hline 245 \\ 3 \end{array}$	x	30	5	7	210	35	<p>_____ is partitioned into _____ and _____</p> <p>First I multiply _____ by _____ Then I add up the products</p>
x	30	5								
7	210	35								
Solve problems, including missing number problems, integer scaling problems,	Using paper folded into 3 to demonstrate 3 times as long/ three times as high		<p>Three times as high, eight times as long $? \times 5 = 20$ $20 \div ? = 5$ 3 hats and 4 coats, how many different outfits?</p>	<ul style="list-style-type: none"> • _____ is _____ times the length of _____ • _____ multiplied by _____ is equal to _____ • _____ times the size of _____ is _____ 						

YEAR 4		Factors factor pairs								
MULTIPLICATION										
Objective	Concrete (representation)	Pictorial (process)	Abstract	Sentence stems						
<p>Grid method recap from year 3 for 2 digits x 1 digit</p> <p>Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)</p>	<p>Show the links with arrays to first introduce the grid method.</p>  <p>4 rows of 10 4 rows of 3</p> <p>Move onto base ten to move towards a more compact method.</p>  <p>4 rows of 10 4 rows of 3</p> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculations 4 x 126</p>  <p>Calculations 4 x 126</p> <p>Fill each row with 126. Add up each column, starting with the ones making any exchanges needed Then you have your answer.</p>	<p>Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles. The different columns to show their thinking shown below.</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1456 678 1680 742"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p>$210 + 35 = 245$</p> <p>Move forward to the formal written method:</p> $\begin{array}{r} 35 \\ \times 7 \\ \hline 245 \end{array}$	x	30	5	7	210	35	<ul style="list-style-type: none"> _____ partitioned into tens and ones is _____ and _____ _____ x _____ = _____ tens x _____ + _____ ones = _____ tens + _____ ones = _____ To multiply a 2-digit number by _____, you multiply the _____ by _____ and the _____ by _____ _____ tens multiplied by _____ plus the ten _____ equal to _____ tens. _____ ones x _____ = _____ ones _____ tens x _____ = _____ tens _____ hundreds x _____ = _____ hundreds _____ tens/hundreds multiplied by _____ hundred from the exchange is equal to _____
x	30	5								
7	210	35								

Column multiplication

Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$

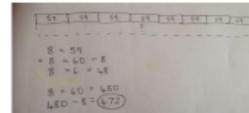


It is important at this stage that they always multiply the ones first.

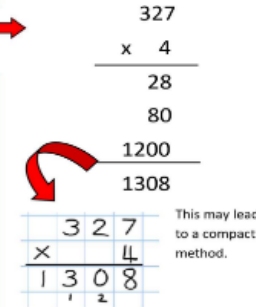
The corresponding long multiplication is modelled alongside

x	300	20	7
4	1200	80	28

The grid method may be used to show how this relates to a formal written method.

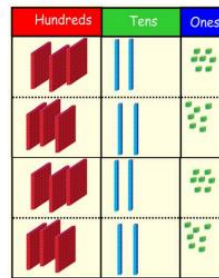


Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



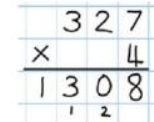
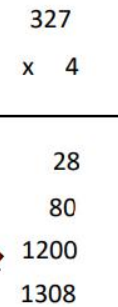
This may lead to a compact method.

Column Multiplication for 3 and 4 digits x 1 digit.



It is important at this stage that they always multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$

x	300	20	7
4	1200	80	28

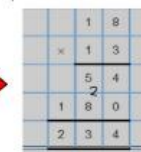


Column Multiplication

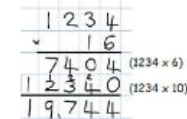
Manipulatives may still be used with the corresponding long multiplication modelled alongside.



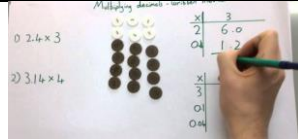
Continue to use bar modelling to support problem solving



18×3 on the first row
 $(8 \times 3 = 24, \text{ carrying the } 2 \text{ for } 20, \text{ then } 1 \times 3)$
 18×10 on the 2nd row. Show multiplying by 10 by putting zero in units first



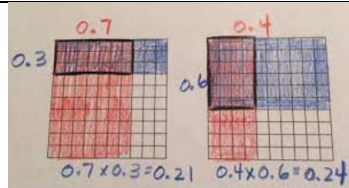
Multiplying decimals up to 2 decimal places by a single digit

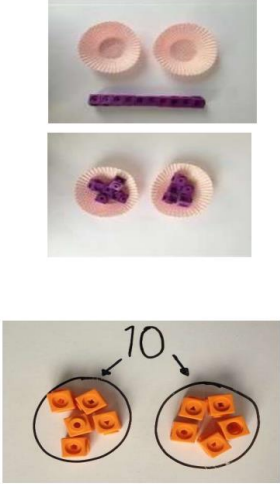

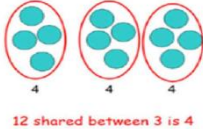
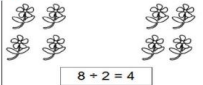
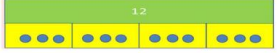


Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.

$$\begin{array}{r} 3.19 \\ \times 8 \\ \hline 25.52 \end{array}$$

Multiplying decimals by a decimal

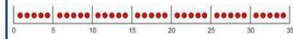
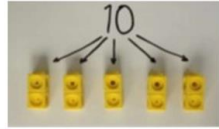


YEAR 1 DIVISION				
Objective	Concrete (representation) Multilink, counters, bead strings, tenframes rekenrek	Pictorial (process) Part whole model Bar model numberline	Abstract Number sentence	Sentence stems
<p>Division as sharing</p>	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p>8 shared between 2 is 4</p> <p>Sharing:</p>  <p>12 shared between 3 is 4</p>	<p>12 shared between 3 is 4</p>	<ul style="list-style-type: none"> The ____ have/have not been shared equally. I know this because ... There are ____ altogether. They are shared equally between ____ groups. There are ____ in each group.
		<p>Children use pictures or shapes to share quantities.</p>  <p>8 ÷ 2 = 4</p> <p>Children use bar modelling to show and support understanding.</p>  <p>12 ÷ 4 = 3</p>		

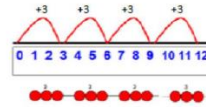
Division as grouping

Divide quantities into equal groups.

Use cubes, counters, objects or place value counters to aid understanding.



Use number lines for grouping



$$12 \div 3 = 4$$


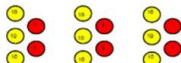
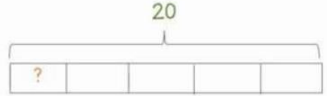
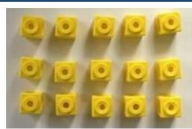
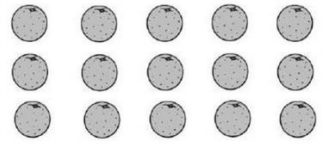
Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.

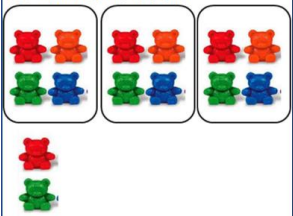
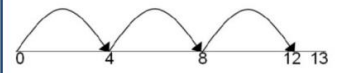

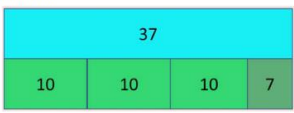
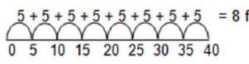
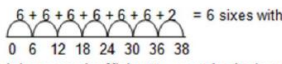


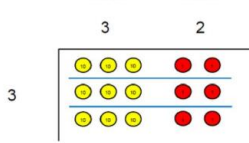
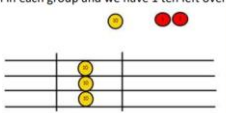
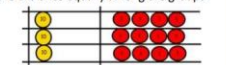
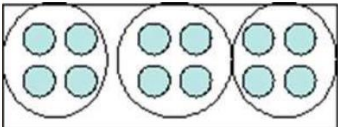
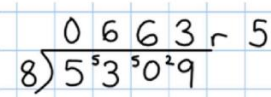
$$20 \div 5 = ?$$
$$5 \times ? = 20$$

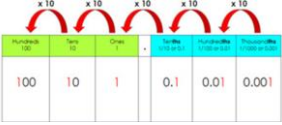
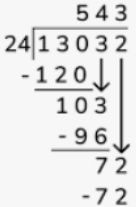
$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?

- The groups are equal/not equal because ...
- There are _____ altogether.
They can be put into equal groups of _____
There are _____ groups.

YEAR 2 DIVISION				
Objective	Concrete (representation) Multilink, counters, bead strings, tenframes rekenrek	Pictorial (process) Part whole model Bar model numberline	Abstract Number sentence	Sentence stems
Division as grouping	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$ 	<p>Continue to use bar modelling to aid solving division problems.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	<p>How many groups of 6 in 24? $24 \div 6 = 4$</p>	<ul style="list-style-type: none"> There are _____ altogether. I have put them into equal groups of _____ There are _____ groups. _____ \div _____ = _____
Division as arrays	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences. $7 \times 4 = 28$</p> $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$	<ul style="list-style-type: none"> There are _____ rows and _____ columns. In this array, I can see _____ \times _____ and _____ \times _____ There are _____ \times _____ = _____ altogether.

YEAR 3 DIVISION		Remainder		
Objective	Concrete (representation) Multilink, counters, bead strings, tenframes rekenrek	Pictorial (process) Part whole model Bar model numberline	Abstract Number sentence	Sentence stems
Division as sharing	<p>$14 \div 3 =$</p> <p>Divide objects between groups and see how much is left over</p> 	<p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p>  <p>Draw dots and group them to divide an amount and clearly show a remainder.</p>  <p>Use bar models to show division with remainders.</p>  <p>remainder: $5 + 5 + 5 + 5 + 5 + 5 = 8$ f 5s in 40?"</p>  <p>mainder: $6 + 6 + 6 + 6 + 6 + 6 + 2 = 6$ sixes with</p>  <p>rs, when it becomes inefficient to count in single mu orded using known facts.</p>	<p>Complete written divisions and show the remainder using r.</p> $29 \div 8 = 3 \text{ REMAINDER } 5$ <p style="text-align: center;"> $\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$ dividend divisor quotient remainder </p>	<ul style="list-style-type: none"> • ____ partitioned into tens and ones is ____ tens and ____ ones. • ____ divided by ____ is equal to ____ • ____ can be partitioned into ____ and ____, as these numbers are both multiples of ____ • ____ divided by ____ is equal to ____ • There are ____ groups of ____ There are ____ remaining. So ____ \div ____ = ____ r ____

YEAR 4-6 DIVISION	Dividend product quotient divisor square numbers cube numbers prime numbers composite numbers			
Objective	Concrete (representation) Multilink, counters, bead strings, tenframes rekenrek	Pictorial (process) Part whole model Bar model numberline	Abstract Number sentence	Sentence stems
Divide at least 3 digit numbers by 1 digit. Short Division	<p>96 ÷ 3 Tens Units</p>  <p>3</p> <p>Use place value counters to divide using the bus stop method alongside</p> <p>42 ÷ 3 =</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p>  <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p>	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder.</p> $\begin{array}{r} 218 \\ 3 \overline{) 4872} \end{array}$ <p>Move onto divisions with a remainder.</p> $\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 432} \end{array}$ <p>Finally move into decimal places to divide the total accurately.</p> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$ 	<ul style="list-style-type: none"> • ____ tens divided by ____ = ____ tens each • ____ ones divided by ____ = ____ ones each • I cannot share all of the tens equally, so I need to ... <ul style="list-style-type: none"> • To use the formal method of division, I start with the digit on the ____ and work from ____ to ____ • There are ____ groups of ____ thousands/hundreds/tens/ones in ____ thousands/hundreds/tens/ones. • ____ hundreds divided by ____ is equal to ____ hundreds with a remainder of ____ • Exchange the remainder, then ____ tens divided by ____ is equal to ____ tens with a remainder of ____ • Exchange the remainder, then ____ ones divided by ____ is equal to ____ ones. <ul style="list-style-type: none"> • ____ ones divided by ____ = ____ ones remainder ____ • When dividing by ____, the greatest possible remainder is ____

<p>Dividing by 10 100</p>	 <p>Using a place value chart and post it notes to move digits along (This is useful because you can stick the post it notes together to model how they move without adding or losing zeros along the way)</p>	<p>Chn to draw own PV chart or use squares in their book to move digits</p>	<p>(see sentence stems)</p> <p>Missing number problems eg:</p> $\underline{\hspace{2cm}} \div 100 = 246$ <p>Or missing operation</p> $845 \underline{\hspace{2cm}} = 84,500$	<ul style="list-style-type: none"> • $\underline{\hspace{2cm}} \div 10 = \underline{\hspace{2cm}}$ • $\underline{\hspace{2cm}} = \underline{\hspace{2cm}} \div 10$ • $\underline{\hspace{2cm}}$ is one-tenth the size of $\underline{\hspace{2cm}}$ • $\underline{\hspace{2cm}} \div 100 = \underline{\hspace{2cm}} \div 10 \div 10 = \underline{\hspace{2cm}} \div 10 = \underline{\hspace{2cm}}$ • $\underline{\hspace{2cm}} \div 100 = \underline{\hspace{2cm}}$, so $\underline{\hspace{2cm}} = \underline{\hspace{2cm}} \div 100$ • $\underline{\hspace{2cm}}$ is one-hundredth the size of $\underline{\hspace{2cm}}$ • $\underline{\hspace{2cm}}$ divided by 10/100/1,000 is equal to $\underline{\hspace{2cm}}$ • $\underline{\hspace{2cm}}$ is one-tenth/one-hundredth/one-thousandth the size of $\underline{\hspace{2cm}}$ • There were $\underline{\hspace{2cm}}$ tens/hundreds. Now there are $\underline{\hspace{2cm}}$ ones/tens.
<p>Long division</p>			$ \begin{array}{r} 1 - 24 \\ 2 - 48 \\ 3 - 72 \\ 4 - 96 \\ 5 - 120 \\ 6 - 144 \\ 7 - 168 \\ 8 - 192 \\ 9 - 216 \end{array} $ 	<ul style="list-style-type: none"> • $\underline{\hspace{2cm}}$ hundreds divided by $\underline{\hspace{2cm}}$ is equal to $\underline{\hspace{2cm}}$ hundreds with a remainder of $\underline{\hspace{2cm}}$. The remainder is exchanged into $\underline{\hspace{2cm}}$ tens. • $\underline{\hspace{2cm}}$ tens divided by $\underline{\hspace{2cm}}$ is equal to $\underline{\hspace{2cm}}$ with a remainder of $\underline{\hspace{2cm}}$. The remainder is exchanged into $\underline{\hspace{2cm}}$ ones. <p>OR</p> <ul style="list-style-type: none"> • $\underline{\hspace{2cm}}$ cannot be divided by $\underline{\hspace{2cm}}$, so there is a $\underline{\hspace{2cm}}$ of $\underline{\hspace{2cm}}$